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THE MECHANICS OF BALLISTIC IMPACT—A SURVEY

T. NICHOLAS

TECHNICAL REPORT AFML-TR-67-208

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THE MECHANICS OF BALLISTIC IMPACT—A SURVEY

T. NICHOLAS

FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division, under Project Number 7351, "Metallic Materials", Task Number 735106, "Behavior of Metals". The research work was conducted in the AF Materials Laboratory, Research and Technology Division, Wright-Patterson Air Force Base, Ohio, by Dr. T. Nicholas of AFML.

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This technical report has been reviewed and is approved.

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ABSTRACT

An attempt has been made to gather under one cover and review the results of a large number of publications pertinent to the field of ballistic impact from a mechanics viewpoint. The major portion of the paper is devoted to a survey of the response of materials to dynamic loading. Structural response and other related problems are also discussed.

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TABLE OF CONTENTS

I. INTRODUCTION

The problem of the analysis and design of materials systems to resist ballistic impact has confronted scientists and engineers for many years. During this time a vast quantity of empirical relation has been accumulated from the results of tests by military authorities interested in armor, but these arbitrary penetration laws apply only to the particular impact conditions employed and generally lack both a logical basis and a physical interpretation. Today, armor systems are given merit ratings based upon their performance in ballistic impact tests but few attempts have been made to relate basic materials properties to systems performance. These highly empirical tests have not added significantly to an understanding of the fundamentals of materials behavior and processes but serve the sole purpose of discriminating between the performance characteristics of various armor systems. It would be desirable, therefore, to approach the ballistic impact problem from a mechanistic point of view and to attempt an understanding of materials behavior under impact loading before trying to analyze the response of complex systems.

When this writer first became interested in the problems of ballistic impact he was confronted with an enormous amount of literature in many diversified fields. The task of searching the literature and finding the relevant sources was aided immensely by the many excellent survey articles and reviews dealing with specific topics which seemed relevant to the problems associated with armor. However, very little could be found dealing specifically with armor problems from a mechanistic point of view and it was felt that a single source should be available to researchers interested in this problem to save them the long and tedious task of searching the literature to become familiar with the subject and focus upon the specific problems requiring current attention. The present report attempts to serve this purpose in anticipation of a continued and growing interest on the part of investigators in this field in the coming years.

The main purpose of this report is to assemble under one cover the results of a large cross-section of the research conducted which is related to the problem of ballistic impact, thus encompassing such topics as strain-rate effects in materials, wave propagation phenomena, shock waves, dynamic fracture, structural response to dynamic loading, and others. The emphasis here is on an understanding of the mechanics of ballistic impact which in turn is based on a fundamental knowledge of materials behavior under dynamic loading which in turn is intimately related to a study of wave propagation phenomena; much of the information contained herein could therefore be of interest to those having

articles which will be listed here for reference. Among the books in the field are the classic ones: by Goldsmith (1), which is one of the most thorough treatments of the broad subject of impact; Rinehart and Pearson (2) on impulsive loading, and Kolsky (3) on stress wave propagation. The book by Kornhauser (4) is a less technical introduction to some of the concepts associated with impact phenomena. Many conferences and symposia have been held during recent years; the technical papers presented at these meetings by the leading authorities in their respective fields can be found in the volumes edited by Huffington (5), Shewmon and Zackay (6), ASTM (7) and the Institute of Mechanical Engineers (8) dealing with materials properties under high rates of loading, by Kolsky and Prager (9) and Davids (10) on stress wave propagation, and by the Air Force (11) on structural dynamics problems. Among the many review articles in the literature dealing with more specific subject areas are the ones by Hopkins (12) and Kolsky (13, 14, 15) on theory and experiments in stress wave propagation; Krafft (16) on some earlier experimental techniques for obtaining high strain-rate data; Braslau (17) on strain-rate dependence in metals; Craggs (18) on plastic waves; Duvall (19) on shock waves, Hunter (20) and Kolsky (21) on viscoelastic waves; Abramson, et al (22) on waves in rods and beams and Hopkins (23) on spherical wave propagation. The reader is especially referred to the recent excellent surveys on dynamic deformations of materials and structures by Cristescu (24) and Hopkins (25) and on impact by Goldsmith (26). Other general references will be listed together with the discussions of the specific topics below.

II. MECHANICAL PROPERTIES OF MATERIALS UNDER DYNAMIC LOADING

A material subjected to stresses through some external loading or testing apparatus undergoes deformations which are related through its mechanical properties or constitutive relations. If the rate of loading is high enough, the inertia forces in the testing equipment may become significant, and if it is even higher, the inertia forces of the specimen itself may become significant; it is in this latter case that a surface disturbance propagates through the body as a stress wave. For rapidly applied loads as are encountered in impact or explosive loading, the mechanical properties of the material can be deduced only by consideration of the wave propagation phenomena occurring within the specimen. However, the analysis of the wave phenomena requires a priori knowledge of the dynamic mechanical properties of the material, which is what is being sought originally. It is this dilemma which makes it difficult to properly interpret experimental data at high rates of strain and which has motivated the search for theories of plastic-elastic and shock wave propagation which are sufficiently realistic in their physical basis and mathematical assumptions. The study of wave propagation thus serves two functions; it attempts to explain the response of materials to high rates of loading and more important, serves as a basis for determining dynamic material properties.

A. Longitudinal waves in bars and rods

A major portion of the experimental work on wave propagation has dealt with longitudinal waves in wires, rods or strips which can be readily instrumented to detect strains on the surface and which can be analyzed relatively simply. The response of materials in the elastic range to impact loading dates back to the nineteenth century and the theory of elastic wave propagation. We can thus find St. Venant (27) relating the stress in a bar to the velocity of an impacting mass in 1868 and John Hopkinson (28) performing the first experiments in plastic wave propagation in 1872 using an available elastic wave theory to explain stress pulses propagating in annealed iron wires. It was not before 1930 however that Donnell (29) studied the effect of a non-linear stress-strain law on the propagation of stress in a bar. He considered a material with a bi-linear stress strain relation and predicted that two distinct wave fronts would propagate through the material with two different wave speeds. There was subsequently little activity in this field until the early 1940's when the first plausible one-dimensional finite amplitude plastic wave theory was developed independently by von Karman (30) in the United States, G. I. Taylor (31) in England, and Rakhmatulin (32) in Russia.

Rakhmatulin investigated the additional complications of unloading waves in a plastic material; an analytic solution was subsequently presented by Skobeev (33). These theories all assumed that the behavior of the material could be described by a single-valued relation between stress and strain in uniaxial stress and that the stress-strain curve was concave towards the strain axis. Radial displacements and their associated three-dimensional effects were neglected. The theories predicted that each level of strain would propagate at its own characteristic velocity given by $\sqrt{S/\rho}$ where S is the slope of the tangent to the stress strain curve at each point and ρ is the mass density. Von Karman and Taylor considered the problem of a constant velocity impact at one end of a specimen and predicted a plateau of constant permanent strain near the impacted end. The analysis of a wave propagating in a rod was extended to a material having a concave up stress-strain curve and the resulting formation of shock fronts was considered by White and Griffis (34). They discussed the behavior under different impact velocities; a similar problem was considered later by Lee and Tupper (35). One of the most significant features of these theories was the assumption of a single-valued dependence of stress on strain, independent of strain-rate, although none of the theories required explicitly that the stress-strain curve for wave propagation should necessarily be the static (or quasi-static) one.

The earliest experimental verification of the Karman-Taylor-Rakhmatulin or rate-independent theory was undertaken by Duwez and Clark (36) who measured the distribution of permanent strain in copper wires which were impacted by falling weights. They achieved good correlation between the permanent strain and the velocity of impact using the static stress-strain curve of copper in the theoretical calculations and proved, in their experimental study, the existence of a critical velocity above which rupture would occur in tension impact as predicted by the theory. Careful examination of the data revealed discrepancies in the measured strain at the impacted end which were explained by imperfect reflections of plastic strain and inaccuracies in the measurement of the duration of impact. On the basis of the experimental evidence they concluded, however, "that the relation between stress and strain under impact conditions differs essentially from that under static conditions".

Karman and Duwez (37) in 1950 reported on the work they had performed during World War II and which had been published originally in classified NDRC reports. They presented the original theory and the results of experiments to verify the rate-independent theory of plastic wave propagation. Using impact loading with a falling weight on copper wires and measuring the permanent strain, they

verified the theory regarding the plateau of constant strain and its relation with the impact velocity. However, they found that the distribution of plastic strain deviated considerably from the theory and concluded that the assumption of a stress-strain curve independent of the rate of strain was not entirely justifiable. Campbell (38) used the plastic wave theory to calculate the dynamic stress-strain relation for copper under tensile impact and demonstrated for the first time that a separate stress-strain curve (not the quasi-static one) could be used in the strain-rate independent finite amplitude wave theory to explain experimental observations.

Further doubts about the validity of a rate-independent theory of plastic wave propagation were cast by the results of experiments on the propagation of incremental stress pulses on material prestressed into the plastic region. Although the theory predicted that a small stress increment should be propagated with the plastic wave velocity $\sqrt{S/\rho}$ determined by the slope of the tangent to the stress-strain curve at each level of strain, Bell (39) observed that an incremental deformation wave always travelled with the elastic wave velocity in prestressed steel bars. In an elaborate series of similar experiments with incremental stress pulses on prestressed copper strips, Sternglass and Stuart (40) observed that the velocity of the wave front was always the elastic velocity and the velocity of any part of the wave was always greater than that predicted by the von Karman theory. They concluded that dispersion or broadening of a stress pulse exists when the material is prestressed into the plastic range and that a theory which neglects strain-rate effects could not adequately describe the propagation of plastic strain. Realizing that these experiments had applied a dynamic stress superimposed on a static prestress, Alter and Curtis (41) performed experiments on a lead bar by applying a dynamic prestress and incremental stress pulse in the form of two closely spaced impacts. However, even in this situation the incremental wave travelled with the elastic wave velocity and it was concluded that a rate-independent theory was inadequate and that a strain-rate-dependent theory would be necessary to predict the experimental results.

Realizing some of the apparent inadequacies of a rate-independent theory, Malvern (42, 43) proposed a theory of longitudinal plastic wave propagation for a material in which stress is a function of both strain and strain rate. He postulated the general form of a constitutive relation as

$$E_o \dot{\varepsilon} = \dot{\sigma} + g(\sigma , \varepsilon)$$

where dots denote time derivatives and $g(\sigma,\varepsilon)$ is an arbitrary function to fit the behavior of a particular material. He carried out calculations for the case when plastic strain rate is a function of the dynamic overstress, i.e., the excess of the actual stress over the stress in a static test for the same value of strain. Mathematically this took the form

$$g(\sigma,\varepsilon) = k\ [\sigma - f(\varepsilon)]$$

where $f(\varepsilon)$ is the stress in a static test and σ the actual stress. This formulation implies that a material is brought to a state of incipient plastic flow after a given amount of elastic strain, independent of the elastic strain rate, but that the plastic flow requires time in which to become appreciable so that the additional strain beyond the static yield strain is mainly elastic. This would explain the propagation of stress increments at the elastic wave velocity in a prestressed bar since time is required for plastic flow to occur. Malvern's calculations did not, however, predict a region of uniform strain near the impacted end of a bar subjected to constant velocity impact as had been predicted by von Karman and verified later experimentally. The rate-dependent theory also predicted an increased maximum strain near the impacted end which was also in disagreement with the early experimental findings. Plass (44) extended the work of Malvern, studying both a linear and an exponential law for dynamic over-stress based on results obtained experimentally for copper and pearlitic steel. He found that the exponential law gave better prediction for larger plastic strains while for smaller strains the two laws were not discernible. Both the rate-dependent and the rate-independent theories were generalized by Lubliner (45) in a theory of plastic wave propagation formulated on the basis of a general quasi-linear constitutive equation. He showed that both theories were special cases of a generalized theory and showed conditions under which one or the other could be valid.

Ting and Symonds (46) studied the longitudinal impact of a viscoplastic rod having a linear relationship between strain-rate and dynamic over-stress. This analysis was extended (47) to a rod whose viscoplastic behavior is described by a power-law relation between strain rate and dynamic over-stress.

There have been numerous experimental investigations in an attempt to verify or disprove the rate-dependence of various metals. Most attention has been confined to aluminum and aluminum alloys, copper, and several other pure metals. Various experimentalists disagree as to a single theory of mechanical behavior under impact loading; the existing theories can be broken down roughly into

three categories: 1) a rate-independent theory using the static stress-strain curve, 2) a rate-independent theory using a single dynamic stress-strain curve, and 3) a rate-dependent theory of the form originally proposed by Malvern (42). There is some disagreement as to whether or not the second formulation can really be called a rate-independent theory because of the use of a single dynamic curve which is different from the static stress-strain curve of the material. The rate-independent theory was formulated on the basis of a single-valued relation between stress and strain; it was not specified what this relation should be or whether or not it should be the static curve. This point was emphasized by White and Griffis (34) and by Bell (48) who pointed out that "nonlinear wave propagation phenomena are essentially different from quasi-static loading, and hence, there is no particular reason to anticipate a priori what explicit form the governing stress-strain curve might possess." For a review and commentary upon some aspects of the existing state of research on the topic of mechanical waves and strain-rate effects in metals see the article by Hopkins (49).

The fact that different investigations arrive at different conclusions regarding the dynamic response of materials can only lead one to question the validity or the proper interpretation of many of the experiments. As has been pointed out earlier, at very high rates of loading it is difficult to measure the stress-strain behavior of a material without studying the stress waves which are necessarily set up, and this in turn requires a wave propagation theory which in turn depends on a knowledge of the stress-strain relation. Contrast the findings of the following experimentalists investigating plastic wave propagation. Ripperger (50) measured the relatively small dynamic strains on long copper bars impacted at one end and found that the strain-rate-dependent theory may predict the strains if the proper value for the strain-rate constant is available and if the nature of the input at the end of the bar is known. At distances greater than five diameters the rate-independent theory was found suitable while in the vicinity of the impacted end neither theory could predict the strains. In subsequent experiments, Ripperger (51) showed that aluminum, copper and iron were all strain-rate sensitive in that they are capable of developing higher stresses than at corresponding strains in a static test. He found that the dynamic yield stress increases with strain rate and that a logarithmic relation between plastic strain rate and dynamic overstress best explained the results. Malvern (52) reported the results of longitudinal wave propagation studies in annealed aluminum bars using two types of measuring devices (an electromagnetic transducer and strain gages) and found agreement with a rate-independent theory based on a single dynamic curve which was slightly higher than the static curve. He concluded, however, that the rate-dependent theory was not invalidated since the two theories predict virtually the same thing for a material with a very slight rate-independent but the rate-independent theory is easier to apply and therefore preferable

in this situation. Bell (53, 54) used an optical diffraction grating technique (55) to directly measure surface strains as high as three percent in constant velocity impact tests on annealed aluminum bars and found the velocity of propagation of each level of strain to be constant and in very good agreement with the propagation velocities determined from the slopes of the static stress-strain curve. He also found the strain levels for various impact velocities to be those predicted from the static stress-strain curve at positions more than two diameters from the impacted end. Deviations within one diameter of the impact end were attributed to large radial accelerations associated with rapidly increasing strain and a correspondingly large dilatation which was observed experimentally. More recently, Bell (48) has generalized his experimental results and has shown that the finite amplitude rate-independent wave theory of von Karman, Taylor, and Rakhmatulin is in close agreement with experiments involving the symmetrical free flight impact of several completely annealed metal polycrystal and single crystal rods. He has found that not only does the finite amplitude wave theory apply to all annealed metals thus far considered, but also there exists an experimentally determined generalized constitutive relation in the form of a linearly temperature-dependent parabolic law which is applicable to both polycrystal and single crystal. (see Ref. (56)). His diffraction grating experiments have shown that in general the governing stress-strain curve for strain-rate-independent finite amplitude wave propagation is not the quasi-static stress strain curve of the material. Sperrazza (57) has verified some of Bell's conclusions in experiments on pure lead bars and found that a single dynamic stress-strain curve gave consistent results using the strain-rate independent plastic wave theory. Contrary conclusions were drawn by Bianchi (58) who performed tests on the propagation of longitudinal plastic waves in long prestressed specimens of annealed copper. He found that the strain-rate-dependent theory in the linear form, and possibly even more in a non-linear form, gives a good description of the whole phenomenon for any value of time, distance from impact end, and static pre-loading. He noted, however, that if the impact velocity rose regularly to a steady state value, the asymptotic behavior was explainable by the rate-independent theory and furthermore, for no static preloading, the rate-independent solution approximated the behavior for any value of time at sections away from the origin. Recently, Bodner and Clifton (59, 60) investigated elastic-plastic pulse propagation at distances far from the impacted end in annealed, commercially pure aluminum bars where small plastic strains governed the material behavior. They found that the general features of a rate-independent theory were verified using a single dynamic stress-strain curve which did not differ appreciably from the quasi-static curve and which exhibited a Bauschinger effect. These conclusions regarding rate-independence seem to apply to most, but not all, of the experimental results dealing

with wave propagation in long bars subjected to axial impacts, yet, there are other considerations in the proper interpretation of these experimental results which must be mentioned. Most of these deal with the assumptions made in the theories of longitudinal plastic wave propagation and can be best analyzed by first considering some parallel developments in the theory of elasticity.

The theory of wave propagation in solid circular cylindrical elastic cylinders is one of the few problems in dynamic elasticity which admits of an exact solution; it is not surprising, therefore, to find no corresponding solutions for materials with non-linear stress-strain curves or plastic materials which can suffer permanent deformations. Because of the complexity of the exact elasticity solutions, many attempts have been made to develop rational approximations. Notable among these is the paper by Mindlin and Herrmann (61) in which a theory for wave propagation in bars was derived in which both radial inertia and radial shear effects were considered. Comparison with the simple one-dimensional theory showed significant differences for very short wave lengths or high frequency components. Skalak (62) solved the problem of an elastic bar impacting a rigid wall at a given velocity using asympototic methods to get numerical results for long times. His solution showed a dispersion of the wave front over an increasing length of the bar as it progressed and a train of sinusoidal oscillations of the strain about its final value behind the wave front. Zachmanoglou and Volterra (63) modified the one-dimensional theory to include additional terms and Miklowitz (64) obtained some approximate solutions to longitudinal impact problems for long times, concluding that the Mindlin-Herrmann theory gave the best approximation and experimental agreement between radial and axial strains. Subsequent investigations have attempted approximate, asymptotic or closed form solutions for various theories attempting to take into account three dimensional effects and using physical considerations to determine the likely nature of the motion (65, 66, 67, 68).

One of the few exact three dimensional solutions in dynamic elasticity has been obtained by Heimann and Kolsky (69) for the propagation of longitudinal elastic waves in thin cylindrical shells. It would be interesting to see if this solution could be extended to the case of a plastic material with subsequent application to the experimental investigation of plastic pulse propagation in thin cylindrical shells.

Finally, a study of the relation between surface strains and average strains was conducted by Graham and Ripperger (70) in longitudinal elastic wave propagation. They concluded that surface strain measurements give a distorted picture for short

wave lengths, the total distortion being dependent upon the pulse shape, and that quartz crystals gave more clearly defined records than strain gages. There is reason to believe that these qualitative findings would be applicable in the plastic case as well; it is this reasoning that has motivated some of the theoretical developments in three-dimensional plastic wave propagation.

Following the idea of assuming a reasonable form for the three-dimensional displacement components, as had been done by Mindlin and Herrmann for the elastic case, Plass (71) included the previously ignored effects of inertia associated with radial expansion or contraction, and of radial shear associated with rapid cross-section changes in a theory of plastic wave propagation in a rod of material exhibiting a strain-rate effect. Lateral inertia effects in plastic wave propagation were likewise considered by Papirno and Gerard (72) and by Tapley and Plass (73) who performed experiments on pure copper. The latter considered a rate-dependent material and showed that the more accurate three dimensional theory gave lower strains at the impacted end of a long bar than the elementary theory of Malvern and thus better experimental correlation. Their work was subsequently extended by Tapley (74) to the case of finite length bars.

In a very extensive study of lateral inertia effects, DeVault (75) has used numerical procedures to solve the strain-rate-independent theory of von Karman for a step impulse loading in the cases with and without lateral inertia effects. He noted that some of the effects predicted by Malvern's rate dependent theory are similar to the effects predicted by a rate-independent theory which includes lateral inertia.

The assumption of a uniaxial stress condition in rods under impact is somewhat questionable; investigations have thus been carried out to examine the three dimensional behavior of impacted rods, especially in the vicinity of the impact. In shock wave studies in plates, measurements of free surface velocities are made in central regions of thin plates impacted by other plates to avoid the edge effects in what is essentially a state of uniaxial strain before edge reflections have arrived. It is apparent then that the initial fraction of a diameter of a rod is in a state of uniaxial strain while portions of the rod more than a few diameters away from the impacted end are closer to a state of uniaxial stress. The transition region from uniaxial strain to uniaxial stress in an impacted rod must therefore take place in the region within the first diameter and must occur through the growth of large plastic deformations behind the initial elastic wave front from reflections from the traction free surface of the rod. In the theory of wave propagation in

non-linear elasticity, Truesdell (76) has given relations for the initial dynamic stresses which arise from contributions of the non-plastic hydrostatic components of uniaxial stress which occur within one bar diameter of the impacted end. Bell (77, 78, 79) has made an extensive study of the development of the plastic wave in the first diameter of an impacted rod using his diffraction grating technique and has shown this behavior to be related to some of the predictions of Truesdell. He has suggested that the apparent effect of strain rate might be a result of the mechanics of the development of the initial plastic wave front. Bell and Suckling (80) have determined the dynamic over-stress and initial peak stresses by measuring the time of contact for various length bars in symmetrical free flight impact of aluminum and have identified the transition velocity between the plastic and hydrodynamic regimes. Ripperger (81) had concluded earlier that measurements near the impacted end were questionable and that surface strain measurements were unreliable near the impacted end.

Another type of experiment for determining the dynamic properties of materials has made use of compression load bars by impacting a softer specimen on a hard bar axially and obtaining the dynamic strains (or stresses) at the impacting face by observing the propagating strain pulse on the hard "load bar" which remains in the elastic region. Kolsky and Douch (82) determined dynamic stress-strain curves for copper, aluminum and an aluminum alloy by firing short specimens at a steel pressure bar and observing the permanent strain in the specimens after impact. They found that the aluminum alloy showed no rate of strain effect while the pure copper and aluminum did, and that the rate-independent theory predicted the permanent strain based on the impact velocity using a single dynamic stress-strain curve. The prediction of the distribution of strain was not as good and repeated impact tests in which the permanent strain was measured after each test did not verify strain hardening theories. These experiments leave doubts as to the actual dynamic unloading behavior of materials stressed beyond the elastic limit. Johnson, Wood and Clark (83) had earlier used a pressure bar technique to relate impact stress to impact velocity and maximum plastic strain. Using the Karman theory they determined a dynamic stress-strain curve which was higher than the static curve and concluded the necessity of a family of dynamic curves based on the final total strain. Sperazza (57) used several measuring techniques including an aluminum pressure bar to further verify the existence of a single dynamic stress-strain curve for large amplitude waves in lead.

A further extension of the load or pressure bar technique for determining the dynamic strains at the end of a bar has been utilized in the now popular split Hopkinson bar experimental arrangement. In this configuration, a relatively short specimen is sandwiched between two load bars which remain elastic. A driver or

ram impacts the first elastic weigh bar and the resulting stress wave propagates down the bar. Part of the wave is reflected at the specimen and part is transmitted through the second weigh bar. The stress waves in the two bars are recorded by the use of surface strain gages. Through the knowledge of the speed of wave propagation through the elastic bars the dynamic stresses and strains at the two surfaces of the sandwiched specimens can be deduced. If it is further assumed that the stresses and strains in the specimen are uniform along its relatively short length, i.e. wave propagation effects can be neglected, a dynamic stress-strain curve can be obtained by averaging the stresses and strains at the ends of the specimen. This "thin-wafer" or split Hopkinson pressure bar experiment was first introduced by Kolsky (84) in 1949 who applied it to the dynamic testing of materials at high strain rates. Kolsky realized the possibility of errors due to inertial stresses and gave an analysis for an inertial correction. Davies and Hunter (85) analyzed the experiment setup carefully, considering the non-uniformity of stress and strain within the specimen and concluded that for length to diameter ratios near unity end friction effects between the specimen and the pressure bars could be eliminated. They concluded, however, that lateral inertia must be taken into account and derived inertial stress corrections from kinetic energy considerations. The split Hopkinson bar technique has received considerable attention in recent years; examples are given by Chiddister and Malvern (86), Davidson et al (87) and Lindholm (88) for determining dynamic stress-strain curves of metals as a function of strain rate. Lindholm (89) has presented details of the technique and presents results for several f.c.c. metals in polycrystalline form and for high purity aluminum single crystals in compression (90), demonstrating a definite strain-rate dependence. Larsen et al (91) investigated the rate sensitivity of single crystal Ag_2Al using a split Hopkinson bar for the higher strain rates and noted that different types of slip mechanisms had different rate sensitivities. They re-examined the theory of plastic wave propagation in the light of deformation rates determining dislocation mechanisms. Karnes and Ripperger (92) modified the experimental technique by including a quartz crystal stress transducer and strain gages at one end of the thin specimen in tests on high purity aluminum and found it to be rate sensitive. They concluded that their experimental technique allowed the measurement of stress and strain at essentially the same point, thereby circumventing the averaging procedure ordinarily used. Hauser, Simmons and Dorn (93) tested thin wafer specimens of high purity aluminum and concluded that its dynamic behavior could not be described by a rate-independent constitutive relation. They neglected the wave propagation phenomena because of the short length of the specimen and attempted to justify this procedure by theoretically checking the adequacy of the approximation by solving the internal wave propagation problem based on an assumed bi-linear stress strain curve and a rate-independent theory which they found to be inadequate. Conn (94)

later re-analyzed the results of Larsen et al using a more general non-linear analysis and demonstrated that the same data could be predicted from a rate-independent theory.

The applications and limitations of a one-dimensional theory of bar impact related to the split Hopkinson bar technique and the effects of wave interactions of elastic and plastic wave components involving loading and unloading stresses are further discussed by Conn (94) who carefully delineates mechanical effects from material properties to be measured. A description of high strain-rate experimental setups is given by Hauser (95) along with a detailed description of the split Hopkinson pressure bar and the equations and assumptions associated with its use.

On the basis of tests covering a wide range of strain rates using conventional testing machines in conjunction with wave propagation experiments and/or pressure bar techniques, constitutive equations have been postulated for several metals and alloys. In addition to the investigations previously cited, Malvern (52) verified a power law relation between stress and strain rate for high purity aluminum based on his experiments with a Hopkinson bar and earlier experiments by Alder and Phillips (96) at lower strain rates. Other constitutive relations for various structural materials are given, for example, by McLellan (97) and Maiden and Green (98). The latter used a split Hopkinson bar similar to the one used by previous investigators (85, 86, 93) for strain rates up to 10^4 on two aluminum alloys and found no rate sensitivity over the range of strain rates tested. This observed behavior was very different from that found for higher purity aluminum by Hauser, et al (93) and Davies and Hunter (85) in which the stress at a given strain was found to increase significantly over a similar range of strain rates. Maiden and Green observed, however, a definite strain-rate sensitivity in titanium which exhibited a delayed yield.

Finally, in contrast to many other workers in the field, Bell (56) has found no rate sensitivity for a large number of metals on the basis of wave propagation experiments using his diffraction grating technique and has found a single temperature-dependent, rate-independent parabolic stress-strain law for all f.c.c. metals. At present there are several efforts being made by researchers to assemble the available stress-strain data from various sources; these should appear in the literature in the near future.

The investigations described above have dealt mainly with materials having relatively smooth stress-strain curves without sharp yield points. There have been several investigations into the behavior of mild steel which exhibits a very sharp yield point and hence cannot be classified together with most other metals. The yield strength of mild steel is known to be highly

rate dependent; Costello (99), for example, has found the ratio of dynamic to static yield strengths to be as high as 2.9 for strain rates of about 10^6 per second using explosive loading techniques. Campbell and Duby (100) have found that steel exhibits a delayed yield phenomenon and have measured this delay time to be between 35 and 80 microseconds. They have interpreted these results in terms of a criterion of yielding based on dislocation theory. Taylor (101) has noted the nonuniform deformation in mild steel at low strain rates in the range of the lower yield stress and has determined the stress required to propagate a plastic zone at different velocities. Taylor and Malvern (102) studied the dynamic lower yield stress and associated plastic deformation and have shown that the effect of a high rate of strain is to increase the resistance of the steel to the propagation of this nonuniform yielding throughout the material, thus confining the local plastic strain to a smaller region at high strain rates. These observations warrant classifying mild steel in a separate category when discussing the rate of strain effects on the mechanical behavior of metals.

One aspect of the wave propagation phenomenon in bars which does not seem to correlate well with existing theories is the propagation of incremental loading waves in bars prestressed into the plastic region. It was the experimental results of Bell (39) and Sternglass and Stuart (40) who observed a small incremental pulse travelling with the elastic wave velocity that led many investigators to conclude that a rate-independent theory of plastic wave propagation was not valid. Karnes and Ripperger (92) have attempted to explain this observed elastic wave velocity in prestressed rods by a rate sensitivity which is representable as a family of parallel stress-strain curves depending on strain-rate. They concluded that to go from the static to the dynamic curve one must follow a path parallel to the elastic modulus. This explanation, however, does not explain the experimental results of Alter and Curtis (41) who superposed an incremental pulse on a dynamically prestressed rod of lead and still observed a velocity of propagation equal to the elastic wave velocity. It must be noted here that the above mentioned experiments dealt with <u>small</u> increments of stress in prestressed bars; Bell and Stein (103) found that large stress increments propagated at the slower plastic wave velocity in agreement with the rate-independent theory. Rubin (104) derived an analytic rate-dependent solution which explained the results of the Sternglass and Stuart experiments. Bianchi (58) also found that a rate-dependent solution was in agreement with his experiments on propagation of waves in prestressed copper specimens, especially for high values of the prestress.

Hunter and Johnson (105) conducted a theoretical treatment of the problem including three dimensional effects and the phenomenon

of geometric dispersion in a rate-insensitive material. The dispersion is due to the neglecting of inertially induced stresses and initially plane sections remaining plain during a transient disturbance. They concluded that geometric dispersion has a profound effect on the propagation characteristics of incremental pulses, and leads in particular to pulse velocities much larger than the plastic wave velocity and comparable to half of the elastic wave velocity, C_o. However, since velocities as large as C_o are not obtained, it is clear that geometric dispersion does not account for the Sternglass-Stuart results and that strain-rate effects appear to be an important factor in these experiments.

An alternative explanation of the incremental wave experiments has been postulated on the basis of the observed Portevin-LeChatelier effect, or repeated yielding in metals. This effect is observed in slow-rate loading tests and is manifested in a stepped or staircase stress-strain curve with distinct jumps. Riparbelli (106) attempted to explain the results of the Sternglass and Stuart experiments on the basis of a discontinuous or stepped stress-strain diagram and proposed that the time derivative of creep deformation (plasticity) is proportional to the excess of stress (a strain-rate effect). The staircase stress-strain curve in pure aluminum has been observed by Dillon (107) in torsion, Bell and Stein (103) in compression, and Sharpe (108) in tension. Kenig and Dillon (109) presented experimental data on the propagation of shear waves under biaxial prestress and have observed catastropic straining at certain "points" in a specimen while other points are not affected. This observation was consistent with the application of the theory of wave propagation to a material exhibiting a staircase relation between stress and strain but no dependence on strain-rate. The wave propagation analysis for the propagation of axial waves was previously given by Dillon (110) in which incremental strain waves were predicted to travel at the elastic bar velocity because the stress-strain relation usually has a local slope equal to the Young's modulus even in the plastic range of deformation. The incremental wave experiment which has often been cited as evidence of strain-rate effects is thus explainable in terms of a rate-independent staircase stress strain curve and can be shown, instead, as Bell (48) notes, "... to be a dynamic experiment related to the study of the quasi-static stability properties of the pre-stress" and "thus provides no information relevant to the applicability of the finite amplitude wave theory."

An alternative approach to the wave propagation phenomenon was presented by Cottrell (111) who discussed deformation at high rates of strain in terms of a dislocation model. He attributed rate of strain effects in metals to the mechanics of dislocation motion and discussed the problem in terms of crystal physics.

Simmons, Hauser and Dorn (112) presented a uniform theory of wave propagation, treating the Karman and Malvern theories as special cases of a more general treatment. They concluded that neither theory is completely realistic for impulsive loading of metals and that further progress in understanding the propagation of plastic waves in crystalline solids is dependent upon the formulation of the laws of motion of dislocations under impulsive loading conditions.

The effect of the rate of strain in metals is still an unanswered question in the minds of many investigators. The results from constant velocity longidudinal impact tests on bars have tended to demonstrate, as von Karman (37) pointed out, that the assumption of a stress-strain curve independent of the rate of strain is not entirely justifiable. Karman realized that the average rate of strain and actual rate differ considerably and pointed out that "it is therefore not logical to utilize tension impact tests to study the influence of the rate of strain on the properties of metals." Lee and Wolf (113) showed analytically that if average values of strain are taken in high speed testing, a spurious strain-rate influence would be deduced when propagation effects first begin to appear as the testing speed is increased. These are not due to rate dependence but rather to strain variations along the specimen. Riparbelli (114), in commenting on constant velocity impact tests, concluded that the stress-strain law must be time dependent to explain experimental results and that the plastic strain exhibited a time lag which could be explained by a plastic flow law.

The multitude of wave propagation experiments still depend on the material constitutive relations for proper interpretation and an a priori assumption as to rate-dependence or rate-independence. The more recent uses of pressure bar techniques (51,52,82,85,88,93) which eliminate the necessity of a wave propagation theory have demonstrated that the dynamic stress-strain curves for most metals differ from the quasi-static ones, the difference being quite small in most cases and even non-existent for some alloys. The conclusions of Bell regarding rate-independence (48, 79) differ from those of his co-workers in the field, although Kolsky (14) feels that "many of these differences are more apparent than real". Bell's work has been based on his diffraction grating technique (55) which has been extended for the measurement of large strains (115). As far as this author knows, this is the only <u>direct</u> measuring technique for observing strains in a wave propagation experiment. The validity of strain gages has been challenged by Bell (116, 117) on the basis of comparison of experiments using wire resistance strain gages in constant velocity impact tests with idential tests using diffraction gratings. "At every position it was found that the wire resistance strain gages gave strains lower than those obtained from the diffraction gratings". Bell found errors in the maximum strain amplitude of 26 per cent at 2.5 per cent strain

and a strain rate of 1000 in/in/sec. In an independent experiment, Malvern and Efron (118) compared results obtained using strain gages with results from identical experiments utilizing a velocity transducer developed by Ripperger and Yeakley (119) based on an electromagnetic induction technique. They observed consistently lower propagation speeds from the transient strain records than those based on the velocity records. Malvern (52) believes that this represents an actual lag in the response of the strain gages and has noted that his velocity measurements are in agreement with those of Bell.

The Hopkinson bar experiment has attempted to circumvent these difficulties and uncertainties by using short specimens where it is assumed that the uniform stress and strain distributions of quasi-static measurements apply during impact. These experiments have been criticized because of the neglect of three-dimensional effects and end friction. Bell (120) has recently examined this experimental configuration by making direct diffraction grating surface strain measurements along short bars sandwiched between two elastic pressure bars. He has observed non-uniform finite strain distributions which he attributes to non-linear wave initiation, propagation, reflection and interaction and concludes that the quasi-static hypothesis is not applicable in the Hopkinson bar experiment. The relative merits of surface strain measurements versus average strain values in this experiment is a question that is likely to receive much attention from investigators in this field for some time to come.

B. Waves of one-dimensional strain

The many problems and uncertainties associated with determining dynamic properties of materials from studies of wave propagation in bars and rods has led many investigators to seek other techniques for determining dynamic materials properties. Most popular and promising among these techniques is the study of plane waves passing through large blocks or plates of a material. In this configuration, the state of strain is essentially one dimensional apart from edge effects because of the symmetry of the deformation which restrains motion normal to the direction of the propagating wave. However, because of this lateral restraint, the stresses or pressures required to cause large plastic deformations are extremely high, often several orders of magnitude above the yield stress of the material.

The speed of propagation of elastic waves in uniaxial strain is governed by the elastic modulus (K + 4G/3) where K is the bulk rigidity modulus and G the shear modulus. The bulk and shear moduli of metals are considerably increased at high pressure and

this leads to the phenomenon of higher stresses propagating more rapidly than lower ones which in turn leads to the front of a propagating stress pulse becoming progressively steeper and eventually almost vertical. This phenomenon is known as a shock wave and is similar to the wave formed in a bar of a material having a concave up stress-strain diagram. A concise discussion of shock waves in solids can be found in Kolsky (9); Skidmore (121) summarizes the basic concepts to supply the physical background for non-specialists in the field.

The first detailed investigation of propagation of shock waves in metals was conducted by Pack, Evans, and James (122) who detonated explosives in contact with steel and lead. They noted that the initial velocity of a shock wave can exceed the elastic bulk wave velocity because of the increase in bulk modulus with pressure and observed this experimentally in lead but not in steel.

The theoretical analysis of waves of uniaxial strain was presented by Wood (123) in 1952 who first stressed the importance of hydrostatic compressibility in determining the nature of a wave. A systematic investigation of wave propagation treating elastic and plastic waves and the formation of shock waves in unidirectional strain was presented by Morland (124) who gave the complete equations for a material having a linear stress-strain relation for low stresses and concave up in the plastic region.

Plane waves of uniaxial strain are commonly generated in flat plates by detonating explosives in contact with the surface or by impacting them with other flat plates which are accelerated to high velocities by explosive means or through gas guns. It is difficult, if not impossible, to observe the wave as it propagates through the plate and measurements are generally confined to the free surface velocities of the back of the plate and the wave speed through the plate. In some of the earliest experiments, Allen (125) used an optical technique to measure surface oscillations on steel plates under explosive loading. Elastic and plastic shock waves in metals were observed by Minshall (126) using electrical pin contactors and crystals. The pin probe technique which he introduced consisted of placing numerous pins near the center of the plate at various distances from the surface. As the plate deforms, the free surface makes successive contacts with the pins and the time of contact is recorded. The pins are spaced such as to prevent reflected waves from the edges or other pins from reaching them before the plane deformation wave arrives, and this information can be translated into a continuous record of free surface displacement or velocity. At about the same time Mallory (127) used the electrical contact technique to study metal-metal impact motions in aluminum; he found that the velocity of

propagation of a shock wave was a smoothly decreasing function of the thickness of target. Allen, et al (128) used a shadowgraphic technique to infer the free surface velocity of a steel plate from the strength of the air shock developed as the wave passed through the plate.

Shock wave measurements can be used to determine equations of state or constitutive relations for a material by assuming some form of theory of wave propagation. In the high pressure region where the induced stresses are much greater than the stresses supported by the cohesive strength, compressible hydrodynamic theory has been employed extensively in the investigation of plane shock waves in solid slabs (19, 129). The theory assumes that the material cannot support shear stresses and treats the material as a compressible fluid to obtain PVT relations for solids at high pressures. In recent years, shock wave measurements have been used extensively together with compressible fluid theory to determine equations of state or Hugoniots of materials at high pressures (129,130,131). The theory assumes a non-linear solid but no strain-rate effects, i.e., stress is a single valued function of strain, and has been used to calculate Hugoniot pressure-density relations for a large number of metals for pressures up to the megabar region (132, 133). While the compressible inviscid fluid type model descrives accurately the overall state of materials at high pressures, yield strength of materials as well as its variation with pressure and temperature must be considered for lower pressures along with certain aspects of wave interactions which may be particularly sensitive to yield strength influences. Fowles (134) performed one of the few experiments in the low pressure region (under 50 K bars) using a streak camera to record shock and free surface velocities. He validated the elastic plastic theory for one-dimensional strain and found no strain-rate effects evident for hardened and annealed 2024 aluminum.

If the pressure volume relation is concave upward everywhere, as in an ideal fluid, a single shock front will form. In a solid, however, yielding produces a discontinuity in the slope of the curve and leads to the formation of two shock fronts, the leading shock, or elastic precursor travelling with a higher velocity for some shock amplitudes. The amplitude of this leading front is directly related to the dynamic yield strength of the material and is called the Hugoniot Elastic Limit (HEL). If the HEL diminishes as the wave progresses through the material, some form of stress relaxation is indicated in the material, i.e., a strain rate effect is present. Duvall (135) shows how the decay of the elastic precursor wave preceeding a shock can be related to material relaxation mechanisms. His equations are formally equivalent to those of Malvern (42) for waves in a bar of a

strain-rate dependent material except for different physical constants because of the difference between the conditions of uniaxial stress and uniaxial strain in the two experiments.

The influence of yield strength on the propagation of plane stress waves has been considered by Lee and Liu (136) who compared an elastic-plastic solution to the hydrodynamic solution and a rigid-plastic solution. The hydrodynamic solution which neglects yield influences showed an appreciable difference in attenuation while the rigid-plastic solution was found to be a poor approximation since the elastic resilience in shear has an appreciable influence on the unloading characteristics. While the determination of the loading portion of the stress-strain curves has been extensively studied (131), very little has been done dealing with unloading in the high pressure region. Hartman (137) studied the unloading curve of shock loaded 6061-T6 aluminum by measuring the residual strains for different shock loads and constructing the curve from a number of points. His comparison of the measured strain with that predicted by the elastic-plastic theory established the inability of the simple theory to predict the release path and he concluded that the Bauschinger effect and an increased flow stress (rate dependence) must be considered. The Bauschinger effect denotes the reduction in flow stress and temporary increase in the work-hardening rate of a plastically deformed metal undergoing unloading. This effect was demonstrated in steel under rates encountered in explosive loading by Jones and Holland (138) using explosively generated plane shock waves. Additional unloading studies were conducted by Baker, et al (139) who determined the unloading behavior continuously and found a deviation in the unloading stress-strain relation from the elastic-plastic theory. They attributed the differences to strain rate effects and Bauschinger effects but noted that the differences were not very great. The plane strain configuration has been used recently by Barker, et al (140) to observe elastic and plastic waves at the free surface with an interferometer technique and a laser light source. Observations of an initial peak in the elastic wave and its subsequent decrease have been related to dislocation motion within the material with the conclusion that the initiation of slip is more difficult than its continuation.

Recent experimental work in high pressure shock waves in metals has been directed towards determining equations of state at increasingly high pressures. Fowles and Isbell (141) have presented experimental results for Hugoniot measurements using a new method for producing very high pressure shock waves. Although the maximum pressures generated by the highest velocity flyer plates in these experiments were only about 2 megabars, this is seen as an initial attempt towards obtaining high pressures than those obtainable with explosives, and much research is being directed in this area.

Contrast these results with those of Russian investigators (142) who have reported experiments on shock wave determination of equations of state at pressures up to 9 megabars but have not given any details of their experimental apparatus.

Another aspect of shock wave propagation at high pressures which has received attention is the metallurgical aspect of the phenomenon. In many crystalline materials including metals and alloys, pressure induced phase transformations, usually temporary but sometimes permanent, are found to occur at various pressures. Many studies have been made of phase transformations, and the work done has been reviewed by Rice, et al (131). A summary of the principal publications dealing with metallurgical effects of shock waves has been presented by Appleton (143) who surveyed studies involving terminal structure and properties of metals and alloys after they have experienced shock waves and pressure induced phase transformations in iron. For a review and description of the most important research applications of shock waves related to the study of behavior of solids see the excellent article by Duvall (19).

The theory of propagation of plane waves of finite elastic and plastic strain and relevant experimental findings has been reviewed recently by Lee (144). A theory of plastic wave propagation which includes elastic and plastic deformation components in the region of finite strain and which also considers thermodynamic influences has been proposed (145) and promises to provide a basis for future investigations in this area.

C. Waves in strings and Cables

The experimental results of longitudinal wave propagation studies in bars and rods have as yet failed to resolve the controversy concerning the rate-dependence on rate-independence of the stress-strain relations of various metals. Of the many uncertainties in this type of experiment, the most pronounced, is the three dimensional effects of radial inertia and radial shear in this assumed uniaxial stress configuration. This problem has been somewhat overcome by reducing the radial dimensions of the specimen; for the case of thin wires this dimension can be reduced by several orders of magnitude from that of a bar or rod. However, as thinner specimens are used, instrumentation for observing longitudinal plastic waves becomes increasingly complex since strain gages cannot be made indefinitely small. Problems of this nature have led several investigators to study the problem of transverse impact on thin wires as a means of determining dynamic mechanical properties of materials.

The study of transverse impact on thin wire and the associated wave propagation in elastic-plastic strings was first considered by Rakhmatulin (146) who assumed that the materials behavior in a strain-rate-independent manner. Assuming a form for the transverse deformation and neglecting flexural rigidity of the wire, he arrived at a closed form solution for the problem of constant velocity transverse impact on a straight wire. The transverse impact problem was subsequently considered by Craggs (147), and experimental data was reported by Ringleb (148) for the impact on linearly-elastic aircraft arresting cables. The mathematics for elastic waves were developed by Li (149) and the theory has been applied by Smith, et al (150) to determine constitutive relations for textile yarns by studying the transverse impact photographically. This experimental configuration has been utilized recently by Schultz (151) to study the mechanical properties of several metals with the aid of stroboscopic photography. By photographing the motion of the stretched wire under constant velocity transverse impact, the rate-independent theory of plastic wave propagation has been verified assuming the quasi-static stress-strain curves and using a closed form solution to the wave equation for an aluminum alloy and for pure aluminum and copper at low strain levels.

An excellent bibliography on studies of wave motion in strings and cables up to 1960 can be found in Ref. (24) and a discussion of some problems is given by Cristescu (152). Finally, the same general theory has been extended to the problem of transverse impact on membranes by Karunes and Onat (153) who obtained a solution highly analogous to the one in the string problem.

D. Torsional wave propagation

An experimental configuration which has a great potential for the generation of meaningful data regarding dynamic materials properties is the case or pure torsion of a circular cylindrical rod. It is usually assumed in the analysis of such a configuration that each cross-section undergoes a rigid rotation; this is strictly true only if the material has a linear stress-strain relation as pointed out by Wolf (154). In general, the solution to the torsional impact problem involves two space variables, but the approximation of dependence on a single space variable is reasonably justified if the curvature of the stress-strain curve is small and a thin-walled tube is utilized. This experimental configuration has been suggested by Craggs (18) for studying the dynamic behavior of materials but has received little attention from workers in the field; the theory of cylindrical wave propagation and its advantages over longitudinal wave studies has been presented by Rakhmatulin (155) and Wolf (154).

Torsional impact tests were performed by Calvert (156) using a flywheel and clutch mechanism and extended to include a static axial tension to determine yield criteria (157). Similar tests by Taylor and Tadros (158) showed a much greater percent increase in both the upper and lower yield point values in torsion than in tension for mild steel. However, none of these investigations presented stress strain curves as a function of strain-rate and no account was taken of wave propagation effects. The only experiments reported on torsional wave propagation have been conducted recently by Baker and Yew (159) using a setup analogous to a split Hopkinson bar in torsion and reducing the data by the averaging process common to that experiment. They concluded that the Malvern rate-dependent theory gave better agreement with their wave propagation results than the rate-independent theory, although they noted from their dynamic stress-strain curves that for shear strains up to 20 percent and strain rates to 2100/sec, annealed copper exhibited significantly lower strain-rate effects than previously obtained longitudinally. It is to be noted that this experimental configuration avoids the three-dimensional effects of longitudinal wave experiments in bars when properly used and deals with shear stresses and strains without the complicating influences of hydrostatic effects. It is felt that much more can be learned from experiments of this type in the future.

E. Other experimental techniques

In addition to the plane stress (waves in rods), plane strain, and pure torsion configurations for determining materials properties several other methods have been employed including some highly empirical techniques as well as some clever experimental configurations for eliminating wave propagation effects. Among the former are the experiments of Davis and Hunter (160) who performed impact indentation tests with a conical tipped projectile. Measurements of indentation dimensions, contact duration, and velocity were compared with hardness measurements and used to assess strain-rate sensitivity of materials in the form of a ratio of dynamic to static flow stresses. Lifshitz and Kolsky (161) made measurements of the coefficient of restitution and time of contact of steel balls impinging on blocks of mild steel to determine the onset of yield and concluded that the procedure was valid for materials with a sharp yield point or for linear viscoelastic materials. Similar experiments yielding empirical relations concerning penetration parameters were reported by Yew and Goldsmith (162) who used hard steel spheres indenting a block of another (softer) material treated as viscoplastic, and by Mok (163) who also used ball-indentations to infer the strain-rate dependence of the yield stress. All of these tests suffer from the difficulty of interpreting the experimental data and correlating them with meaningful physical parameters or deriving materials properties from them.

Another type of experiment has involved the impacting of flat ended cylindrical projectiles against relatively rigid blocks and studying the permanent deformation of the projectile to determine dynamic yield stresses. Lee and Tupper (35) have presented a complete analysis of the problem based on elastic-plastic theory; Raftopoulos and Davids (164) have derived a direct numerical analysis of the problem for several different stress-strain relations. The latter have obtained good experimental correlation for mild steel assuming a rigid work hardening behavior in this type of experiment which was first proposed by Taylor (165) and Whiffin (166) in 1948.

An alternative approach to the plane strain configuration for shock wave propagation studies has been the generation of spherical waves with explosives. Among the works of this kind, Theocaris, et al (167) have recently generated low pressure shock waves in perspex spheres, analyzing them using the hydrodynamic equations. They employed a Moire' method to measure the shock wave velocity and evaluated the particle velocity behind each shock directly from the Moire' pattern. Lifshitz and Kolsky (168) have made experimental measurements of the change of shape of explosively detonated spherical pulses to determine the bulk modulus in linear viscoelastic materials. They found bulk losses to be much less than those in shear but a constant fraction of the latter and concluded that the same microscopic dissipative processes occur in both cases.

One of the more promising techniques for determining the dynamic tensile properties of materials is the use of radially loaded rings and cylinders to study the hoop (tensile) stresses without the complications of wave propagation effects. This method was employed in 1950 by Clark and Duwez (169) using hollow cylinders pressurized from inside; they found that the ultimate stress and proportional limit both increase with strain rate in steel. More recently, Ensminger and Fyfe (170) have studied the cylindrical wave generated by an exploding wire in a hollow cylinder of aluminum. Although these experiments were only exploratory, they served to verify the constitutive relation for an aluminum alloy in the elastic range and showed promise of broader applications in determining materials properties. A similar configuration has been used by Niordson (171) who tested specimens in the form of thin rings. In his setup, he used a magnetic field to expand the ring rapidly and photographic methods for measuring strains. This type of experiment shows promise of generating meaningful data in the future.

Among other new experimental techniques, mention must be made of the experiments of Frasier and Karpov (172) on hypervelocity impact in wax.

They have employed an induction wire technique to obtain experimental data on loads and deformations in the interior of a target with wires imbedded within. Although the material investigated (wax) is not of great interest, it is felt that much is to be learned from this type of experiment on the actual form of wave propagation within a material as it is related to the behavior on the free surfaces.

F. Dynamic multiaxial testing

While a great deal of emphasis has been placed on the determination of materials properties in one-dimensional stress or strain configurations, little attention has been paid to determining dynamic properties under three-dimensional states of stress. This is understandable in lieu of the difficulties that are encountered in even the simplest states of stress as has been pointed out in the previous pages, hence most experimental work has concerned itself with uniaxial tension or compression testing. However, a few attempts at multiaxial testing have been reported. Hickel, et al (173) have reported results of burst tests on cylinders (a bi-axial state of stress) and have attempted to correlate these data with results from dynamic uniaxial tests on sheets to determine dynamic failure criteria. Gerard and Papirno (174) determined dynamic stress-strain relations from expanding thin spherical diaphragms. Their dynamic bi-axial testing was limited to one σ_2/σ_1 ratio because of the specimen geometry and there was no direct comparison of the bi-axial data with uniaxial data on the same material. Lindholm (88) has presented some preliminary results for dynamic behavior of steel from bi-axial tests in combined tension and torsion using a newly developed pneumatic machine (175) with a capability of applying dynamic loads ranging continuously from pure tension to pure torsion. In addition, at present there are preliminary experiments being performed at several laboratories with the aim of developing dynamic bi-axial loading devices. These experiments promise to lead to more meaningful data on materials behavior under complex states of stress and should lead to the development of generalized dynamic stress-strain relations in three-dimensions and associated yield and failure criteria in the future.

III. Behavior of Materials Systems

In this section an attempt will be made to outline a class of impact problems which have been treated analytically. The problem of the response of structural configurations to dynamic loading is of great importance and covers an extremely wide variety of problems. Dynamic loads can be classified depending on the shape and time of the applied stress pulse or the mass and velocity of the impacting projectile. The response of the system will depend on many factors including the materials properties and geometry of both projectile and system, impact velocity and others. The subject will be broken down arbitrarily into several categories, depending on the type of response or failure of the system, although it must be pointed out that these categories are by no means unrelated and the distinction between response of materials and of structues is not easily defined. Thus, the problem of spalling could be considered as a materials property as well as a type of structural response; likewise it cannot be treated independently of wave propagation phenomena. Finally, emphasis is placed on response of systems where strength mechanisms are important; little attention is paid here to hypervelocity impact phenomena and cratering studies where analytic techniques are not well developed. It is to be noted that only a few simple structural impact problems have been treated in the literature. The difficulties of incorporating non-linear and time-dependent mechanical properties into already complex three-dimensional structural analyses can be well appreciated; the existence of solutions for only simple structures thus reflects these difficulties. The solutions to a wide variety of impact problems can be found in the excellent book by Goldsmith (1).

A. Large dynamic deformations under impulsive loads

This type of problem is categorized by a short duration pressure loading over a wide area of the structure, typically a conventional blast type loading. The significant feature of this type of loading is that the structure does not undergo significant deflections until the loading has decayed to zero; the impulse has the net effect of transferring a momentum to an initially undeformed structure. One of the earliest solutions to an impulsively loaded plate is by Hopkins and Prager (176) who calculated the response of a simply supported circular plate to a uniformly distributed rectangular pulse loading. Wang and Hopkins (177) considered a similar problem for a built-in circular plate under an ideal impulse, i.e. applying a constant velocity to the plate at $t = 0$. The material was assumed to obey the Tresca yield condition and the associated flow rule. Wang (178) later extended the solution to a simply supported plate. The problem of blast loading on a square plate

was treated by Cox and Morland (179) for a simply supported plate of a rigid plastic material under an ideal impulse loading. As in the previous solutions, bending stresses were assumed to predominate and membrane and shear stresses were neglected. Ellington and Ellington (180) considered both bending and membrane stresses for the solution of a clamped circular plate under an ideal impulse. They developed an approximate solution by assuming the form of the deflection curve and the position of a radially moving plastic hinge and determined critical impulse values for failure to occur. Witmer, et al (181) developed a computer code for determining the large deformations of a wide class of structures under impulsive loading. The material was assumed to have a stepwise linear rate-independent stress-strain curve and good correlation was obtained with experimental results of explosively loaded structures (182). Florence (183) presented a rigid plastic analysis of a clamped circular plate under a uniform pressure loading applied instantaneously and decaying in time. Using the Tresca yield condition he found the permanent central deflection and its dependence on pressure and impulse. This work was extended to a plate loaded by a rectangular pressure pulse uniformly distributed over a central circular area (184); as in most of the other solutions only bending action was considered. As still another problem in impulsive loading, Sarkar (185) considered a rectangular plate of a linear viscoelastic (Kelvin) material under a concentrated impulsive load applied at a point on the plate. This analysis, as with most of the others cited, is rather limited in application because of the simplified constitutive equations used; more realistic equations which consider the non-linearities and time dependence of real materials must be used if good correlation is to be expected between experimental and theoretical results in blast loadings on simple structures.

B. Wave propagation in beams and plates

When structures are subjected to impact by high velocity projectiles, a disturbance is propagated through the structure. This type of problem has been studied for impacts on beams, plates, and other simple structures to determine the stresses and deflections. One of the earliest investigations into transverse impact on beams was conducted by Duwez et al (186) for a long beam impacted at the free end with a constant velocity. A moment curvature relation was assumed based on the (static) stress strain curve of the material; it was observed that the strain in the beam was not propagated at constant velocity. The concept of rigid plastic behavior and of plastic hinges in beams under impact loading was introduced by Lee and Symonds (187) to study the large plastic deformations of beams under transverse impact. Parkes (188) applied an ideal rigid plastic analysis to the dynamic problem of a cantilever impacted at its free end and achieved satisfactory correlation with experiments for the prediction of permanent deformation at points remote from the impact. Ting (189) extended the work of Parkes to include

a strain-rate sensitivity and found good experimental correlation. Bodner and Symonds (190) conducted impact experiments on mild steel and aluminum beans to evaluate the rigid-plastic assumptions and found that a strain-rate dependent rigid-plastic analysis gave excellent agreement with experimental results. The effects of shear on the dynamic plastic deformation of beams was studied by Karunes and Onat (191) using a Timoshenko beam type analysis. Symonds (192) presents a recent review of theories and experiments for analyzing plastic deformations of metal structures under impulsive type loads.

The propagation of waves in a membrane under transverse impact was studied by Karunes and Onat (153) using a rigid strain-hardening model for the material. The solution was found to be analogous to that of the transverse impact of a wire.

The problem of a rigid cylinder impacting a plate was treated by Bakhshiyan (193) by considering only the shear stresses in the plate in a visco-plastic analysis. A similar problem was treated by Kochetkov (194) using a slightly different stress-strain relation to find the stresses, strains, and velocities in the plate in addition to both the elastic and plastic shear waves in the plate. Cristescu (195) considered the problem of the impact on a plate by a circular rigid cylinder which also possesses a rotational motion and determined the axially symmetric plastic waves in the plate.

The problem of a disturbance propagating radially outward from a circular hole in an infinite elastic plate has been treated by Scott and Miklowitz (196, 197). Using the exact equations of elasticity they have solved the cases of a step normal displacement in the radial direction and a radial body force with step-time conditions. Davids and Lawhead (198) analyzed the propagation of stress waves from a unit step impact pulse at an oblique angle in an infinite elastic plate with a thickness of the order of a wave-length. Pytel and Davids (199) studied analytically the normal impact on an elastic plate with a concentrated load. The solution describes the stress wave propagation through the plate and leads to a study of scabbing or spalling which will be discussed below. In addition to these studies of compressional waves, the flexural waves travelling in beams and plates have been analyzed by Davids and Koenig (200) using a "direct method" of numerical integration. Finite elements and the associated boundary conditions have been considered and it has been shown that maximum stresses can occur after reflection from boundaries for certain impulse loadings. Miklowitz (201) has analyzed the flexural stress waves in a plate due to a dynamic concentrated load considering both the effects due to bending and to shear and rotatory inertia.

C. Scabbing or spalling

The phenomena of scabbing or spalling was first observed in 1914 by B. Hopkinson (202) while investigating the effects of explosives. Spalling is a dynamic tensile fracture caused by a high intensity, short duration compressive stress pulse being reflected at the free surface of a body as a tensile stress wave. The phenomenon, which has also been referred to as Hopkinson fracture, is intimately related to the propagation of stress waves through a material and to the dynamic tensile fracture strength of the material. The portion of the body which is separated due to the fracture is termed a scab or spall. The type of waves which can cause scabbing are ordinarily generated from explosive detonations near the surface of a material or by the high velocity impact of projectiles. Broberg (203) studied the plane shock waves reflecting normal to a free face to determine the criteria involved in spalling. His experimental results could not be closely correlated with theoretical predictions because of a lack of knowledge of the mechanical properties and strength at high rates of strain. From his experimental data he also reasoned that fracture did not occur instantaneously under the loading rates encountered in shock wave propagation. Other investigators have presented qualitative descriptions of the spalling phenomena but little has been achieved in the way of good quantitative predictions because of the aforementioned lack of knowledge concerning dynamic materials properties. Davids and Kumar (204) have presented a survey of scabbing and an analysis of wave propagation by graphical techniques. Herrmann et al (205) presented analytical methods of predicting uniaxial stress waves and spall and discussed the technique for studying spall using plate impact experiments. A detailed analysis of the effect of an air cushion between impacting plates was given. Rinehart (206) presented a survey and general discussion of the concepts of spalling and has extended the concepts to multilayered materials and to layers of different materials. For a set of solutions to a large number of problems connected with fracture occurring under impulsive loads and some basic rules for the solution of problems of this type, the reader is referred to the work by Rinehart (207). Several interesting features of the theory of propagation of transient dilatational waves as related to some previously unexplained features occurring in scabbing and the problem of spalling are presented by Hunter (208). Among the findings is the relation between displacements far from an impulsive load and the pressure-time profile of the load.

D. Plug formation in plates

Another type of failure which may occur when a projectile strikes a plate is the formation of a plug. In this failure mechanism, the projectile tends to push an approximately cylindrical

portion of the plate ahead of it, although the "plug" may or may not be separated from the plate. This phenomenon is characterized by a shear failure in the plate along an approximately cylindrical surface through the thickness of the plate. In most analyses of this type of problem, only shear stresses in the plate are considered. In the aforementioned solutions of Bakhshiyan (193) and Kochetkov (194) the impacting cylinder was assumed to have an infinite mass; these solutions were thus not able to explain plug formation but rather the disturbances away from the region of the plug. Chou (209) approached the problem of perforation of plates by high speed projectiles analytically assuming visco-plastic behavior of the plate material. By assuming a shear failure he was able to predict the size of the perforated hole and achieve qualitative agreement with experiments. However, as is often the case in simplified analyses of this nature, he concluded that "since the theory involves numerous simplifying assumptions as well as estimated properties of the materials, its accuracy in predicting the hold diameter is doubtful. A realistic theory must include the compressibility, viscosity, and heat conduction properties of the material". Pytel and Davids (210) studied plug formation failure by analyzing a plate given a uniform initial velocity over a circular area from an impact. Under the assumption of viscous behavior of the material in shear they achieved good correlation with experimentally observed deformations. Minnich and Davids (211) obtained a direct numerical solution to the plug problem using viscous-plastic shear strengths and determined the dynamic yield strength in shear of a steel armor plate. For higher velocity impacts and the resulting plug formation and penetration Thomson (212) applied a visco-plastic flow theory for the solution of hypervelocity perforation of thin plates; by including the target material yield strength in the solution he found markedly different velocities, displacements and stresses than when the yield strength was neglected.

E. Penetration of plates

A large body of literature exists on the penetration of plates by high speed projectiles of various types. Most of this literature is somewhat empirical in nature and does not consider analyses of plug formation, spalling, etc. but seeks rather to define penetration criteria in terms of various parameters such as momentum and kinetic energy of the projectile, target density or hardness etc. The question of which parameters are of greatest importance is answered by attempting to relate experimental findings to empirical predictions. No attempt will be made here to describe in detail all the works in the field but mention will be made of a few typical works of this nature. The reader is again referred to

Goldsmith (1) and Cristescu (24) for a review of the literature on dynamic plate penetration.

In the area of analytical analyses of armor penetration can be found the work of Thomson (213) giving a quasi-dynamic approach to the problem. Assuming that the projectile penetrates and that the hole size is large compared to the thickness of the plate, he derived equations for the energy dissipation due to plastic deformation and heating of the projectile-target interface. Zaid and Paul (214, 215, 216) conducted a series of studies on perforation of plates by several types of projectiles. They studied the normal impact of a conical projectile from a momentum viewpoint using a simple model and achieved good experimental correlation (214). This momentum approach was later extended to truncated conical and ogival projectiles assuming the formation of a plug without deformation of the surrounding material (215). Again assuming perforation, they further extended their work to include truncated cones striking the plate at oblique angles and obtained a complete velocity-displacement history (216). They concluded that "inertia effects predominated over material strength at high velocities". Recht and Ipson (217) have derived analytical equations defining the dynamics of ballistic perforation for blunt and sharp-nosed fragments impacting plates at various angles. They can predict the residual velocity of fragments that have perforated a plate using a penetration model and can predict the minimum perforation velocity (ballistic limit) using a plug formation type of analysis. Goldsmith et al (218) presented an analysis of normal elastic impact and perforation at minimal velocities of thin aluminum plates due to flat faced plastic and conical hard steel projectiles. In these and other experiments it has been generally noted that plugs form in hard thick plates, dishing and petaling occur in thin ductile plates and ductile hole enlargement and spalling occur in softer thick plates. Frictional adhesion between a spinning projectile and a target during ballistic penetration has been measured with a torsional-type Hopkinson bar by Krafft (219) who found that sliding friction accounts for about three percent of the striking energy of the projectile.

Mahtab and coworkers (220) have conducted research on cratering and present a theory for dynamic indentation involving dimensional analysis. Their results for projectiles of differing cone angles indicate a possible use of dynamic flattening experiments to examine the dynamic behavior of a material. They assume in their analysis that the diameter of the permanent crater formed under impact is a function only of impact velocity and mass, static indentation pressure, and density of the target. This is just one example of a vast amount of empirical research being conducted dealing with cratering; other references are to be found in the literature.

In addition to studies of projectile-plate impact at ballistic velocities, a great deal of work has been conducted in recent years dealing with hypervelocity impact. It is not the intent of this

paper to consider this subject area although some work of this nature should be cited for reference. Maiden, McMillan, and Sennett (221) have conducted a theoretical and experimental program on thin sheet and multiple sheet impact by hypervelocity projectiles. Riney and Heyda (222) have carried out hypervelocity impact calculations in an attempt to correlate them with experiments involving the impact of a cylinder into a plate. For further research in this area the reader is referred to the proceedings of the several symposia on hypervelocity impact.

IV. Related Problems in Impact

A. Thermal effects

Most research on behavior of materials and systems under impact has neglected the thermal effects which are present to some degree in all impact phenomena. Because of the short duration of the processes of deformation and flow, it is usually reasonable to restrict analyses of wave propagation and impact phenomena to the isothermal case. This assumption is usually justified for slow processes within the range of elastic behavior of a material, where temperature changes are small; however, larger temperature changes may be expected when the material undergoes plastic deformation because of the dissipation of energy. Observable macroscopic temperature increases have resulted from local temperature changes in the vicinity of slip planes. Because of this heat generated, Dillon (223) has found it necessary to consider the coupling between the thermal and mechanical fields and has considered the conservation of energy in improving isothermal plasticity with a coupled thermoplasticity theory. Dillion (224) has studied the heat generated and the rates of heat generation and plastic work from slow torsional oscillations of annealed copper tubes into the plastic region. The observations of temperature are made under nearly adiabatic conditions and have been found to be in good agreement with a more generalized thermoplasticity theory. The experimental technique used for measuring small temperature changes within the linear range of material response is presented by Dillion and Tauchert (225) and shows excellent agreemtn with linear thermoelastic theory.

Recht (226) has studied the heat generated during dynamic deformation and catastropic shear, which occurs when local temperature gradients offset the strengthening effects of strain-hardening. Experiments on steel and titanium via the machining of specimens have shown that titanium and its alloys are particularly sensitive to catastrophic shear; it has been noted that catastrophic slip is an influential deformation mechanism during ballistic impact.

On the subject of theories incorporating thermal effects, Fine and Kraus (227) present the dynamic behavior of a medium according to an uncoupled thermoplastic theory and compare it to the results from an uncoupled quasi-static analysis. They have solved the problem of a constant theat input at the boundary of an elastic, perfectly plastic medium to illustrate the role of the inertia terms in the solution. Lee and Liu (145) have developed relations for waves of plane strain including both finite elastic and plastic strains and have included thermo-mechanical coupling effects in a generalized theory, although no explicit solutions have been presented.

B. Other materials

The research described in this report dealing with behavior of materials and systems under impact loading has dealt with pure metals and alloys almost exclusively. The reason for this is the attempt at reproducibility of data from investigation to investigation. In recent years, new materials have been developed for specific applications and some of these have found their way into use in armor systems. Foremost anong this new breed of materials have been the fibre or filament reinforced composites. With little a priori knowledge of their dynamic properties some of them have found application in energy absorbing systems such as lightweight armor although they have originally been designed specifically for strength purposes. Gupta and Davids (228) conducted an experimental study of the penetration resistance of fiberglass-reinforced plastic against small-caliber projectiles and found that their weight efficiency was better than that of steel in stopping a projectile. McMillan et al (229) conducted hypervelocity impact studies on tubular stainless-steel targets armored with several types of internally reinforced beryllium including rings of mesh and uniformly dispersed filaments. In this case, however, the reinforced targets showed little reduction in damage compared with the unreinforced targets. In yet another study, Stepka (230) evaluated the impact-fracture characteristics of liquid-filled tanks with walls of several filament-reinforced plastic materials and compared them with those of a aluminum alloy tank. Under impact of projectiles at velocities up to 6500 ft/sec., one of the composites was most resistant to fracture damage while another composite was least resistant. These studies indicate some of the potential applications of composite materials, but a knowledge of the mechanics of their dynamic behavior is required before they can be intelligently designed and applied in structural systems under impact.

Up to now, little research has been conducted dealing with dynamic materials properties of composites, although several investigators are known to be involved in this new area at present. Abbott and Broutman (231) conducted an experimental study of wave propagation in a filament reinforced composite and calculated an apparent dynamic modulus on the basis of surface strain gage measurements. Plunkett and Wu (232) studied the attenuation of plane waves in a composite bar having a thin viscoelastic layer connecting a cylinder to an outer serrated ring.

The attenuation or dispersion of stress waves in fiber reinforced composite materials makes them likely candidates for application in structural systems where spalling is a problem. This desirable mechanical property can be attributed to the use of fibers having elastic moduli much higher than those of the matrix material. This impedance mismatch in a multiphase system has been

studied in the simple situation of plane waves in layered media. Zvolinski and Rykov (233) have studied the reflection and refraction of an incident plastic wave at an interface of two different materials. Kinslow (234) studied the stress waves through a material composed of alternate layers of two different materials. A layered material was found to have much better resistance to spalling than a homogeneous material if the alternate layers had grossly different moduli. The same principle is felt to apply in both fibre reinforced or particulate filled composites if the filler and matrix are of different materials.

A limited amount of research has been conducted dealing with dynamic properties of fibrous materials. Coskren and Chu (235) have performed impact experiments on glass fibre and other webbing materials to determine their impact and energy absorbing characteristics. Petterson and coworkers (236) have studied textile materials under high-speed impact and the forms of energy absorbtion involved therein. They found that 40% of the energy of a bullet is absorbed by the transversely displaced portion of the fabric in motion. Other forms of energy dissipation were the strain energy in transverse motion kinetic energy of fibres, and heat. It is felt that further research on dissipation in fibres under impact and their proper incorporation into composite materials systems may lead to better and more efficient structural systems to resist high speed impacts.

Another class of materials for which little is known regarding dynamic mechanical properties is ceramics or brittle materials. The dynamic mechanical testing of these materials is one of the difficult problem areas because of the small strains involved at fracture. Abbott and Cornish (237) have used a stress wave technique for determining the dynamic tensile strength of aluminum oxode. No rate dependence was noted from measurements with foil type strain gages up to strain rates of 100 in/in/sec. Sedlacek (238) has determined the tensile strength of alumina using hydraulically expanded cylindrical test specimens, and although a rate-dependence has been reported, the results are scattered and highly uncertain.

In the subject area of wave propagation in materials it is to be noted that nearly all the efforts dealing with non-linear and inelastic stress wave propagation have been confined to one-dimensional cases. An exception to this is the work of Aggarwal et al (239) who have investigated wave propagation in two-dimensions using a bi-linear plasticity theory. They have solved the problem of a plane wave up to the elastic limit scattered by a rigid cylinder and have used a bi-linear stress strain curve for both the deviatoric and dilational parts of the stress tensor.

C. Numerical techniques

The mathematics involved in the analysis of problems involving non-linear and inelastic materials and structures is highly complex and often unwieldly; it is therefore not surprising to find that the class of problems which have been solved involve idealizations and simplifications of the actual physical situation. Thus, in the study of materials behavior, materials are often classified as either non-linear but time independent or time dependent but linear; materials with yield points are often treated as elastic-plastic or rigid-plastic. To combine the actual properti s into complex structural systems leads to mathematical difficulties often beyond the scope of current analytical capabilities. Some of these difficulties have been circumvented through the use of high speed digital computers on which calculations can be carried out numerically for highly complex problems. Baron et al (240) have developed a particle in cell code for impact or blast problems involving large displacements and velocities when the behavior of the material is essentially hydrodynamic. Davids and Mehta (241) have developed a direct numerical method for analyzing stress wave propagation in solid bodies by incorporating the governing physical laws directly into the computer program. This program has been used to study cylindrical and spherical waves in an elastic medium and the solutions obtained compare favorably with known analytical solutions (242). Riney (243) has performed numerical calculations for hypervelocity impact problems by considering only hydrostatic stresses using a visco-plastic model for the material. Wilkins (244) has developed a computer code to handle calculations including elastic and plastic components of stress in the resulting difference equations. Ting and Symonds (245) have developed a numerical technique for analyzing impact on finite visco-plastic rods when the strain rate is a non-linear function of stress and strain.

Finally, in the area of dynamic response of structures, Witmer and coworkers (181) have developed an elaborate set of computer programs to analyze the elastic-plastic response of a large variety of simple structures to an impulsive type load where the materials properties can be represented by a stepwise linear stress-strain curve.

D. Research in the future

After examining this large body of literature related to the problem of ballistic impact, the question arises as to what our present day capability is for analyzing an impact phenomenon and what information is lacking in our understanding of terminal ballistics.

Continuing efforts in developing more sophisticated three-dimensional computer codes promise to lead to better correlation

between analysis and experiment when more realistic information regarding materials properties is programmed into the computer. This in turn puts a greater demand on the materials engineer for developing more realistic three-dimensional constitutive relations and equations of state. The questions of rate dependence in metals, the validity of surface strain versus average strain measurements in a bar, the forms of the unloading curves in dynamic testing, and the three-dimensional dynamic response of materials are all likely to receive continuing attention.

The entire field of the dynamic behavior of composite and multiphase materials is relatively unexplored. Much work in the micromechanics or "microdynamics" of composites is necessary to explain their behavior under impulsive loading and the dispersion of waves in such materials. Because of the potential value of these materials in resisting spall and the current development of three-dimensionally reinforced composites, this area can be expected to receive much attention.

The metallurgical aspects of high speed impact and the relations between dislocation motion, plastic deformation, and continuum theory are other subjects that can be expected to receive additional attention in the future.

Finally, the biggest and most important problem will be the combining of knowledge from the many diversified fields and arriving at analyses of ballistic impact phenomena which consider the mechanical, thermal, and metallurgical aspects of the problem and which are sufficiently realistic in their assumptions to be able to relate to ballistic testing. It is the bridging of the wide gap between fundamental research on dynamic behavior of materials and analytical analyses of the dynamics of structural systems on the one hand and actual ballistic testing on the other that must be accomplished. If this approach is realized, it is to be expected that the design of new and better armor systems can be put on a more rational and intelligent basis rather than depending on the present day trial and error type of approach.

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13. ABSTRACT

An attempt has been made to gather under one cover and review the results of a large number of publications pertinent to the field of ballistic impact from a mechanics viewpoint. The major portion of the paper is devoted to a survey of the response of materials to dynamic loading. Structural response and other related problems are also discussed.

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